

A mathematical explanation of Quantum Mechanics

The standard model of Quantum Field Theories is the most sophisticated mathematical description of physics, but also includes some difficult acceptable characteristics, like spin and energy proportional to a detected frequency. I try to explain so-called elementary particles and all their characteristics on a different manner, such that all elementary particles and all their characteristics emerge from this mathematical symmetry analysis. All elementary particles must be described mathematically as point particles rotating in the two-dimensional plane orthogonal to the observed direction of motion. This mathematical, i.e. linear, analysis follows from Einstein's theory of General Relativity. It is based on Einstein's (Comprehensive) A(ction) P(inciple), i.e. that curvature of space and time should always be included in any mathematical description of our reality. Spin, energy connected to elementary particles proportional to a detected frequency, the statistical difference between bosons and fermions and the reason for more particle families of fermions are explained with this mathematical analysis. The theoretical model is not yet finished. The Bell experiment still has to be explained using this mathematical description of orthogonal extended particles. Also the mass relation of the three different families of fermions still has to be explained on simple mathematical principles, but up to now the possible explanation of QFT appears promising! In the following article a short overview of the pre-requisites of this model explaining QFT is given.

The use of spin in SR QM and Q(uantum)F(ield)T(heories)

In all well-known quantum theories spin, i.e. intrinsic angular momentum, is used in a not understood manner. In all quantum mechanical analyses, elementary particles are described as point-particles, which are mathematically described on a linear S(pecial)R(elativistic) so-called worldline. Angular momentum implies extendedness, because momentum in space outside the worldline itself is assumed. In all QM models, including QFT with so-called second quantisation with creation and annihilation operators, spin is explained using a mathematical state-function in complex Hilbert-space, which describes the possible mathematical position of the quantum particle using quantum statistics.

All QM analyses are relativistic only analyzed S(pecial)R(elativistic). A G(eneral)R(elativistic), i.e. including a mathematical analysis of curvature of 4D-spacetime due to gravitation (interaction between always attracting masses), analysis has never been taken into account in any QM analysis. In the past ten years much interest has been focused on so-called string theories, but up to today, 2D-, or so-called Super-, String theories have never been tested experimentally. Instead, the so-called Standard Model of QFT {QED (photon) and the weak nuclear forces (Z and W[±] gauge bosons) mixed via the Weinberg angle appear in the U(1)×SU(2) gauge symmetry and all hadrons which follow from the last possible gauge symmetry group SU(3) of the only possible 4D-space-time universe, together with all elementary fermions, i.e. leptons, as last possible sources of all bosons, which mathematically describe all force-fields between all massive, and possibly also electric charged, particles (all fermions and all bosons, except the spin2 graviton and the spin1 photon)}, is the most trustworthy description of physics today.

Instead of including (the up to now still not understood) gravitational field in the standard model, masses of so-called elementary point-particles are taken into account in the SR QM analysis through the introduction of a so-called Higgs field. In all currently analyzed string theories (2D-Super String theories) it is also assumed that an elementary spinless Higgs boson exists, together with its Super Partner, the so-called Higgsino!

This Higgs-field exists according to the Super-Symmetric SM out of extremely heavy (> 170 GeV/c² according to the last measurements at the Tevatron Collider) spinless elementary particles.

Einstein solved gravitation, i.e. curvature of 4D-spacetime, using the mathematical work of Bernard Riemann. In this work curved, i.e. not-linear, mathematical problems are solved in a linear so-called Riemann-space. This is the curved space described in a higher dimensional mathematical, i.e. linear, space such that the standard method of mathematics can be used again. Einstein called the amount of needed degrees of freedom in this linear Riemann-space $N > 4$. Also see [1], chapter 6.

A simple linear mathematical analysis of curvature of space-time induces $N = 2 \times 4 = 8$, also see analysis [2].

According to Einstein his C(omprehensive)A(ction)P(rinciple), see [1] chapter 30, every mathematical description of our reality must always also include curvature of space-time. None of the currently used QM models, including the SM of QFT, complies with Einstein's CAP. The five 2D-SuperString theories comply with the CAP, however these theories require a 9D-space, i.e. a 10D-spacetime. This space does not allow fermions and as a direct result of that is not valid to describe our, or in fact any possible, universe! Even when 6 spacelike degrees of freedom are contracted into a so-called Calabi-Yau space, these degrees of freedom remain available and as a result of that mathematical fact do not allow a mathematical description of fermions with possible knots in their paths.

The main question now is, how should QFT be rewritten, such that this mathematical description is in compliance with the CAP?

First of all a more thorough understanding of so-called elementary particles will appear very useful:

As David Hilbert in 1900 on the international Mathematical congress at Paris proposed in problem 6 of his then offered 23 problems in math- and physics, also see [3], it would mathematical be beautiful if all characteristics of everything we experience every day could be derived mathematically from a geometrical symmetry analysis.

Such an analysis appears to be possible.

First of all I want to mention that all GR transformations on infinitesimal level can be analyzed linear, i.e. SR. Curvature of space-time now implies two consecutive steps instead of the one dimensional step in linear space-time. Also see [2]. Curvature of space-time implies a doubling of degrees of freedom.

Let us first of all analyze all possible infinitesimal, i.e. linear, transformations of a 4D-space-time vector. From this mathematical analysis it follows that the only possible space in which fermions can be described is our well-known 4D-space-time universe. I.e. any description of our, and any possible other universe, must use a mathematical 4D-space-time description with the degrees of freedom doubled.

All possible infinitesimal transformations of a 4-vector are given with the most general 4 x 4 tensor with 16 degrees of freedom. This tensor must for all possible transformations comply to the maximum speed of massless particles. This is an easy applied demand. This transformation tensor $T_{\mu\nu}$ can be represented in only one way as the sum of a symmetric and an anti-symmetric tensor:

$$T_{\mu\nu} = S_{\mu\nu} \oplus A_{\mu\nu} \tag{1}$$

In the search of non-reducible representations of all possible infinitesimal (i.e. linear) relativistic transformations of a 4-vector the symmetrical and anti-symmetrical transformation groups can also be represented non-reducible with {boson \otimes fermion} groups:

$$S_{\mu\nu} \oplus A_{\mu\nu} \text{ is 1 to 1 representable as: } \{\text{spin}2 \otimes \text{spin}1/2\} \oplus \{\text{spin}1 \otimes \text{spin}1/2\} \tag{2}$$

The symmetrical transformation group is represented 1 to 1 by spin $1/2$ masses and their resulting spin 2 gravitational field, which describes the always-attractive mass-interaction.

The anti-symmetrical transformation group is represented 1 to 1 by spin $1/2$ charges and their resulting spin 1 EM-field, which describes the EM-charge-interaction, represented non-reducible by the spin 1 photon.

As is generally well-known the EM-field equations of Maxwell describe the EM-field completely after applying the U(1) Lorentz-gauge symmetry. The maximum allowed gauge symmetry in any 4D-spacetime universe just is the above described gauge symmetry: $U(1) \times SU(2) \times SU(3)$ (3)

In this mathematical symmetry analysis all elementary particles with all their characteristics slowly begin to appear in a linear mathematical analysis.

How should all QFT, with all used point-particles, be adapted such that at infinitesimal (local linear SR) level the mathematical description complies to Einstein's CAP!?!

For each described degree of freedom, the only possible mathematical doubling of degrees of freedom in our 4D-spacetime universe (reality) is a mathematical extendedness in the 2D-plane orthogonal to the analyzed degree of freedom. In the most general case one is able to describe curvature using a SR analysis, because curvature of space-time can be analyzed SR at infinitesimal level. Choose the direction of motion, i.e. the worldline of a particle, as the z-axis. As long as the particle exists it will now, described as an exact mathematical point-particle, oscillate harmonically in the 2D-plane orthogonal to the z-axis. The harmonic oscillation in the 2D-(x,y)-plane, or in polar coordinates 2D-(ρ, φ)-plane, can be solved exactly with two consecutive (eigen-)time derivatives. The bracketed eigen-time is the time measured by the observer at rest at the average position of the orthogonal extended particle, just as the word eigen-time is always used in QFT. The solution space has all the characteristics of the Hilbert space of QFT. The average extendedness of a CAP orthogonal extended massless particle is:

$$2\langle \rho \rangle = (3/2)^2 \text{Phi} \frac{sc}{\omega} \tag{4}$$

With: Phi = $1/2(\sqrt{5} + 1)$ the golden ratio, s the spin (conserved helicity) of the (massless) elementary particle, c the speed of light and ω the angular frequency.

Expression (4) shows the following characteristics:

1. A spinless elementary particle is only allowed to move along its 1D-worldline.
2. As the frequency becomes zero: $\langle \rho \rangle \rightarrow \infty$, so just as characteristic 1 the mathematical point-particle again moves along a straight line, i.e. doesn't carry any energy or just is fiction.

In the case of a massive particle $c \rightarrow v(\rho, \partial_\rho \varphi, \varphi, \partial_\rho \varphi) < c \wedge \text{Phi} \rightarrow \text{constant } C > \text{Phi}$, such that the characteristics found for massless particles remain valid for all possible massive particles.

From these characteristics it appears obvious that *elementary* spinless particles, like the assumed elementary spinless Higgs boson, are fiction!

The observer moving with the elementary particle and describing it in simple mathematical, i.e. linear, language from the inertial frame with origin at the average position of the 2D-extended particle seems in this simple mathematical analysis much larger compared to the so-called Planck-length. However, keeping in mind that SR Lorentz-contraction of a particle moving with the speed of light (spin1 photon and only other massless spin2 graviton) which cannot be observed extended by the human eye because a particle moving with the speed of light always as a result of that results in a not-observable extended particle, easily sees that this expression is experimentally completely acceptable.

The described extendedness in the 2D-plane orthogonal to the observed direction of motion explains spin and energy of the described elementary particle:

The harmonic oscillating point, which describes for example a photon, has conserved quantities enforced by symmetry laws. The time-component appears to be the energy $H = \mathbf{h}\omega$, with \mathbf{h} Dirac's constant (Planck's-constant divided by 2π) and ω the angular frequency. On the other hand the space-like constant appears to be the conserved angular-momentum in the direction of motion of the harmonically oscillating photon and this is the always conserved helicity \mathbf{sh} . *N.B. the time- and space-like-constants in this case of course do not form a relativistic 4-vector!*

In all QFT the photon is described by a point, moving along its, linear SR or curved GR worldline, and this point describes all characteristics of this 1D-moving point using mathematical vectors fields which represent all the well-known characteristics of this photon, like the EM-field and the spin (with observed 2 degrees of polarization). A description of the QM photon in compliance with Einstein's CAP implies that the photon as exact described mathematical point (-particle) should be described as an harmonic oscillating point-particle in the 2D-plane orthogonal to the observed direction of motion specified by the so-called relativistic worldline. According to the D(ifferential)E(quations) of this SR oscillation, any possible elementary particle is never present on the average position on the worldline itself, but harmonically oscillates around it in the 2D-plane orthogonal to the worldline.

To be mathematically exact a minimum distance of the point describing the harmonic oscillating photon and all its characteristics has a minimal distance with respect to its worldline given by:

$$\rho_{\min} = \frac{1}{2}\rho_{\max} = \frac{3}{4} \frac{sc}{\omega} \Phi = \frac{1}{2} \text{ times the average extendedness given in (4)}. \quad (5)$$

The two consecutive first order DE from which the harmonically oscillating motion in this 2D-plane is derived require 2 B(oundary)C(onditions) to be able to solve the mathematical solution completely.

These BC are either open, or closed. In the case of closed BC interactions in the 2D-plane of oscillation are not possible. Such particles as a result of that fact are able to be on exactly the same space-time location with exactly the same quantum state. Besides, of such particles only one type related to every possible symmetry group following from the symmetry groups (2) and (3) exists. This shows that all bosons, i.e. force particles, should be described with closed BC.

The same mathematical analysis shows that all possible fermions must be solved with open BC. These solutions have one additional quantum number in the form of an integer number > 0 . This quantum number specifies the so-called particle family of all the allowed fermions following from GR symmetry laws. The open BC imply mathematically also that fermions interact in all possible spacelike directions. As a result all fermions comply with Pauli's exclusion principle.

The fact that fermions interact in all directions also implies why they always have masses > 0 . Besides that, fermions are always also allowed to have charge related to the anti-symmetrical symmetry-group of (2). A direct consequence of this conclusion is that all charged particles must have mass.

The fact that fermions are massive implies that in any arbitrary SR analysis knots are possible in the harmonically oscillating path of the vibrating point-particle. As Grisha Perelman proved in 2004 knots are only possible in 3D-space, i.e. 4D-space-time, also see [4]. With this result, I'll return to the first assumption, that first of all, easy imaginable 4D-space-time would be analyzed, in the end it now automatically appears, that 4D-space-time in which any possible universe can be described using simple mathematics are the only (mathematically described) universes possible.

From this completely relativistic symmetry analysis all experimentally detected elementary particles with all their characteristics can be described and understood completely based on mathematical (linear) symmetry

analysis. Besides this beautiful characteristic of this symmetry analysis, using a non-reducible description, it also becomes clear at once that other not yet detected elementary particles are not possible on non-reducible mathematical grounds.

From the short described explanation of all used QFT (Hilbert-space, spin, energy, mass, charge, etc.) it also appeared that the SU(3) quarks must be described with 4 degrees of freedom. From this mathematical fact it appeared to me in 1998 that quarks are NOT elementary spin $\frac{1}{2}$ fermions with additional so-called isospin, but must be elementary spin $\frac{3}{2}$ particles without so-called isospin. Just remember that all possible characteristics of all possible elementary particles should follow from the complete non-reducible relativistic symmetry analysis. From the most general infinitesimal set of symmetry groups (2) it appears that only the spins $\{2, 1, \frac{1}{2}\}$ are stable identities/particles in any possible universe. In this way it is shown at once that quarks, as (real) elementary spin $\frac{1}{2}$ fermions cannot be observed on their own, without a surrounding combined quarks-sea surrounding them. To me, this is the main reason why protons and neutrons of our 3 family's universe are very stable. Such combined so-called baryon-particles are always kept together by the strong nuclear force, which is represented by a so-called gluon sea, which keep the three quarks of a baryon together as combined spin $\frac{1}{2}$ fermions. So, of combined gluons and mesons more different families of bosons are possible, but the only real reason is the fact that these bosons are build out of more particle families of fermions (quarks), of which always more particle families are possible.

Taking everything together, only the following elementary particles are possible in our 3-fermion family's universe, as specified in table 1:

All possible elementary particles in our 3-fermions families universe:

Fermions: <i>3 different families</i>	Bosons:
Leptons: electron, muon and tauon + anti-particles	Graviton, the spin 2 elementary massless boson
Leptons: massive uncharged neutrino's + anti-particles	Photon, the spin 1 elementary massless boson
Quarks 1 st family: up-quark and down-quark	Weak nuclear forces: spin 1 massive elementary gauge-bosons W^{\pm}, Z
Quarks 2 nd family: charm-quark and strange -quark	Strong nuclear forces: spin 1 colored quark+anti-quark gluons
Quarks 3 rd family: top-quark and bottom-quark	Mesons: all non-gluon boson-quark combinations

All fermions have so-called **anti**-particles with sign inverted charge of charged particles and sign inverted helicity of uncharged particles. All elementary leptons are spin $\frac{1}{2}$ particles and all elementary quarks are spin $\frac{1}{2}$ particles.

Tabel 1 All possible elementary particles, as derived from a non-reducible infinitesimal GR geometrical symmetry analysis.

The total amount of elementary fermion families of our universe is 3. Using the given GR analysis below, other universes with different amounts of elementary fermions are of-course also possible:

The total amount of energy absorbed in a black hole of a certain different universe (different physical constants, which imply they can't interact, or so-called "see" one another), until a by these natural constants determined GR mathematical singularity occurs, determines mathematically the amount of energy available to the Big Bang of the new (for example our) universe (with new physical constants). The total amount of energy, together with the direct at the start of the Big Bang created new physical constants, determine the amount of possible elementary fermion families of the new, for example our, universe. The same mathematical process of a Big Bang is the only mathematical explanation of any possible 4D-space-time universe. Constants of Nature in principle have arbitrary values, and this implies that any two different universes will always have no chance at all to interact, i.e. feel, touch or see one-another. In this mathematical analysis the following sequence occurs: energy consuming Black Hole \rightarrow Big Bang at the moment of first mathematical singularity \rightarrow A Big Bang, with energy of the singularity of the Black Hole available as all the energy of this new universe.

The amount of elementary particles families will, during the complete time of existence of any possible universe, be a constant number.

To finish of this simple explanation of QFT I still want to mention the following:

At this moment, string theorists (Edward Witten, Graham Greene, R. Dijkgraaf, Jan de Boer and many other scientists) try to include gravitation in an extended QFT. There are 5 different 10D-space-time theories, which were shown by Edward Witten, using his proposed 11D-space-time M-theory, are mathematically equivalent in that they can be transformed into each other using symmetry transformations. I.e. the 5 different mathematical Super String theories describe the same reality from different viewpoints. However, the so-called “Super String” theories appear mathematically correct *only* if Super Symmetry also is valid.

Super Symmetry is a symmetry between fermions \leftrightarrow bosons. According to this symmetry every elementary particle has a “super-partner”. The graviton is assumed to have as super-partner the spin3/2 so-called gravitino. In this analysis it is assumed that only one type of gravitino particle exists. This implies only one kind of family of super-symmetric fermions and not 3 families of fermions as in the case of the normal observable partners of these assumed “super-partners”. This is easily proven mathematically to be non-sense! In the case of charged lepton families {electron, muon, tauon} it is assumed that the super-partners are spinless and these spinless super-partners of bosons suddenly appear in 3 families {selectron, smuon, stauon}. My simple mathematical analysis of QFT shows that super-symmetry is not valid in any 4D-space-time universe. First spinless elementary particles don't comply with the CAP and secondly because of all bosons only one type of elementary particle exists, while of all possible elementary fermions we have 3 different particle families in our universe.

From this point of view, the required validity of Super Symmetry removed all trustworthiness and validity of the world-famous, but up to this day now still not experimentally proven “Super String” theories.

Measurements at the LHC proton accelerator in the old LEP ring of CERN will decide whether the elementary very massive spinless Higgs boson exists, and after that a search will be started to detect the lightest super-partner(s), i.e. test validity of Super Symmetry. This is the only reason why the *extreme* expensive LHC collider is build!

According to the S(tandard)M(odell) and the experimentally found masses of all elementary particles during the last Tevatron measurements the mass of the Higgs boson $m_H < 147 \text{ GeV}/c^2$. String theoreticians assume the Higgs mass $m_H > 170 \text{ GeV}/c^2$ [5], because they assume validity of super-symmetry with all Super Partners with even higher masses compared to the heaviest top-quark. The fact that the Higgs boson is not been found up to $170 \text{ GeV}/c^2$ is seen by string theorists as the first experimental sign that Super Symmetry is valid. Only if the Higgs mass exceeds about $1 \text{ TeV}/c^2$, the SM with spinless Higgs boson must conflict with observations of our universe. The observed deep-space of our universe shows a very flat universe, while the presence of a very heavy Higgs field implies a more curved universe because the Higgs field should be everywhere to explain the masses of all massive particles. Besides this not understood QM analysis, the only possible reason of mass must be explained from symmetry characteristics. From this point of view, the only reason for so-called mass is interaction with the spin2 gravitational field of all massive particles. Mass due to interaction with a very heavy spinless Higgs-field contradicts this complete geometric symmetry analysis, *and* also cannot be deduced from Einstein's theory of GR!

In this analysis I try to explain Special Relativistic Quantum Mechanics and Quantum Field Theories of the Standard Model on simple linear mathematical grounds. In the now assumed successor of the SM, i.e. the Super String theories with assumed super-symmetry, QM is used not understood ad-hoc. This is why to me all usage of the word “Super” has lost its mathematical logic validity! Quantum Mechanics, i.e. quantized elementary particles theory, implies mathematically a complete non-reducible symmetry-group analysis. All characteristics of these symmetry groups specify all characteristics of all possible elementary particles in every possible 4D-space-time universe completely. In fact, QFT and SR QM are not that difficult to understand and follow completely from Einstein's theories of relativity (SR and GR). It's a pity to me that I've never had the opportunity to meet Albert Einstein in person to discuss this analysis with him!

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Used work:

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