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2. Two-dimensional extended elementary particles must be solved with two B(oundary)C(onditions).

This is a not yet finished first written explanation!!!

According to Einstein's CAP, [1] chapter 30, any valid description of physics must include the gravitational action, i.e. describe curvature of 4D-spacetime. A simple mathematical (linear) analysis shows that all so-called elementary particles must be described linearly as harmonic oscillating points in the 2D-plane orthogonal to the observed direction of motion (given by the SR worldline). This is explained in point1, i.e. in footnote [2].

The exact D(ifferential)E(quation) to be solved for all possible elementary particles in any mathematical possible universe (with required 4D-spacetime) is given by:

$$\rho'' + \frac{k\,\rho}{(H - ½k\rho^2)}(c^2 - \rho'^2) - \frac{c^4L^2}{(H - ½k\rho^2)^2\rho^3} = 0 \qquad (2.1)$$

This equation, as given is the DE of a massive particle, but the DE in the case of the massless graviton and photon, is easily extracted from this most general DE.
DE's (2.2), (2.4) and (2.5), also in polar coordinates, give the complete ***<u>massless</u>*** solution:

$$\rho'' + \frac{(\partial U/\partial\rho)}{(H - ½k\rho^2)}(c^2 - \rho'^2) - \frac{c^4L^2}{(H - ½k\rho^2)^2\rho^3} = 0 \qquad (2.2)$$

In which the potential energy is chosen to result in harmonic oscillation: $U(\rho) = ½k\rho^2$ (2.3)
This result (2.3) to describe harmonic oscillation exactly is used in DE (2.1).

$$\varphi' = \frac{L}{\{(H - ½k\rho^2)\,\rho^2\}} \qquad (2.4)$$

$$z' = z = 0 \qquad (2.5)$$

The speed of the harmonic oscillating point in the 2D-plane orthogonal to the worldline now is the constant speed of light c, i.e.: $\rho'^2 + \rho^2\varphi'^2 = c^2$ (2.6)
Inserting (2.6) into (2.4) yields for the massless graviton and photon the relatively easy to solve:

$$\rho'^2 = c^2 - \frac{c^4L^2}{(H - ½k\rho^2)^2\rho^2} \qquad (2.7)$$

With ρ the polar distance from the observed average position (given with the SR worldline) of the harmonic oscillating particle. The extendedness is described from the inertial frame with origin moving with the particle on its average position on the SR worldline. The CAP required extendedness results in the exact pointlike position of the particle to be oscillating harmonically in the 2D-plane orthogonal to the worldline. With respect to the chosen inertial frame the harmonic oscillating motion represents a constant energy H and a 3D-vectorlike angular momentum **L**, directed in the observed direction of motion specified by the SR worldline, and with in (2.1) up to (2.7) used absolute value L = |**L**|. Constant k is the force-constant to describe the force centered to the average position (origin of the chosen inertial frame) of the extended particle, and needed to yield harmonic oscillation of the oscillating point in the 2D-plane orthogonal to the SR worldline, see (2.3). Constant c is the well-known lightspeed and accentuation marks represent time derivatives of the observables with respect to the inertial-frame. I.e. this equation of motion (2.1) is a second order DE, which requires 2 BC to be solved exactly. Only the solutions of the extendedness of the two massless bosons can be simplified to easy to solve first order time DE problems, as given in (2.7).

Equations (2.1) up to (2.4) cannot be solved exactly, however an exact set of two consecutive first order DE's of the polar distance squared ($x \equiv \rho^2$) can be solved exactly. Again two BC are required to solve this consecutive set of two first order DE's in x exactly.

The general DE to be solved for all possible (2D-extended) elementary particles is:

$$c^2t^2(x) = A\,\frac{\{((6f_m x/A+(f_m)^2-4\,f_m+1+3C/A^2)^2+3(1+3C/A^2-f_m)^2)(2((1+f_m)^2+3C/A^2-3f_m x/A)\}}{(36c(f_m)^2)^2\{(3(1+3C/A^2+(f_m)^2)+i\sqrt{3}((f_m)^2+1+3C/A^2))(4B/(9A^3)-(1-C/A^2)x/A+2(x/A)^2-(x/A)^3)\}}\ *$$

$$*\ \{2((f_m)^2-f_m+1+3C/A^2)F(z_m,\chi_m)-(3(1+3C/A^2+(f_m)^2)+i\sqrt{3}((f_m)^2-1-3C/A^2))E(z_m,\chi_m)\}^2 \tag{2.8}$$

With:

$$A = 2H/k\ [m^2] \wedge B = (3cL/k)^2\ [m^6] \wedge C = (2m_0c^2/k)^2\ [m^4] \wedge \underline{\text{I.e. Mass}}\text{: } (\tfrac{2}{3})^2B \rightarrow (\tfrac{2}{3})^2B + Cx \tag{2.9}$$

Constants A, B, C and a possible charge specify the elementary particle completely. A specifies the particles energy $H = hf = \mathbf{h}\omega$ $[kgm^2s^{-2}]$, with h Planck's constant and **h** the reduced constant of Planck. f the frequency of oscillation in the 2D-plane orthogonal to the observed direction of motion and ω the angular frequency. B specifies the square of the intrinsic angular momentum in the direction of motion, i.e. the conserved spin in the direction of motion via $L = \mathbf{h}s$, with s the spin of the elementary particle. Constant C specifies the rest energy, i.e. mass m_0, of the described extended elementary particle. All fundamental quantities of all particles are divided by the force constant k, as shown in (2.9). I.e. an exact expression of k, which depends only on fundamental quantities of the described elementary particle, is needed to solve (2.8) completely!
The proper time of the harmonic oscillating extended particle (2.8) depends on incomplete elliptic integrals of the first ($F(z_m, \chi_m)$) and second ($E(z_m, \chi_m)$) kind. It should be noted that time t in (2.8) is the time with respect to the inertial frame, which moves with the harmonic oscillating (exact point-)particle with origin at the average position of this harmonic oscillating point. This origin is the actual SR point at which the elementary particle is assumed in all QFT, i.e. it's the particle's proper time.
The appearing "fraction" of the massive elementary particle is given by:

$$f_m(A,B,C) = \{2\beta + \gamma + \sqrt{\{4\beta(\beta-1)+\gamma(12\beta-\gamma(3-\gamma)^2)\}}\}^{1/3} \tag{2.10}$$

With dimensionless constants:

$$\beta = \frac{3B}{A^3} \wedge \gamma = \frac{3C}{A^2} \tag{2.11}$$

In the massless case the easy solution of both the graviton and the photon returns:

$$f = f_{m\equiv0} = f_m(A\,,B\,,C=0) = \{2(\beta+\sqrt{\{\beta^2-\beta\}})-1\}^{1/3} \tag{2.12}$$

In the massless cases the only possible value of fraction f is: $f = Phi = \tfrac{1}{2}(\sqrt{5}+1)$ (2.13)
In this case constant β (2.11) has the same value: $\beta = f = Phi$ (2.14)
So, both basic constants ("fraction" f and $\beta = 9c^2L^2k/(8H^3)$) now are equal to the well-known Golden Ratio!
The massless solution (2.14) results in both arguments $z = z_{m=0}$ and $\chi = \chi_{m=0}$ of the integrals to be such that the integrals exist: $|z| < 1 \wedge |\chi| = 1/\sqrt{2} < 1$ (2.15)
The mass contributions γ in (2.10) will in all possible relativistic situations (for all possible elementary particles) be such that all used incomplete elliptic integrals are valid (including the third incomplete elliptic integral needed in the solution of polar angle φ):

$$|z_m| < 1 \wedge |\chi_m| < 1 \tag{2.16}$$

The (first two) arguments of all incomplete elliptic integrals are:

$$z_m = \sqrt{\{\tfrac{1}{2}(1+i(6f_m x/A+(f_m)^2-4\,f_m+1+3C/A^2)/(\sqrt{3}(1-(f_m)^2)))\}} \tag{2.17}$$

$$\chi_m = \sqrt{(2/\{1+i\sqrt{3}\frac{(1+(f_m)^2)}{(1-(f_m)^2)}\})} \tag{2.18}$$

With fraction f_m given in (2.10).

Both BC have two possible solutions:

- **<u>Open BC</u>**. In this case, the oscillating motion repeats itself after more complete circles of the polar angle φ. The amount of complete circles, before the harmonic oscillating motion repeats itself can be viewed as the quantum number specifying the family of the described fermion in compliance to Einstein's CAP. Our universe has 3 different particle families of fermions, which only differ in rest mass of the elementary particle of the described family. This is a direct result of the longer harmonic-oscillating path with interaction with the gravitational field, before the harmonic oscillating path repeats itself. The constant L in the appearing DE's is the helicity of the CAP extended elementary particle. If L = 0, the elementary particle isn't extended anymore. If L =0, i.e. a spinless boson, the DE's must be solved with closed BC.

- **Closed BC**. In this case, the only allowed interaction is in the observed direction of motion (worldline). These solutions describe all so-called bosons. Only solutions with closed BC allow massless elementary particles (spin1 photon and spin2 graviton). The solution of spinless bosons results into average extendedness equal to zero. I.e. such particles aren't extended, and always move on their 1D SR-worldline. As a direct result of this mathematical fact, elementary spinless bosons do ***NOT*** comply with Einstein's CAP. I.e. such bosons are only fictitious particles, which are invented by physicists, who do ***NOT*** understand Einstein's theory of GR.

The given DE's (2.1) up to (2.7) describe elementary particles in compliance with Einstein's CAP. In this case, a so-called elementary particle isn't described as a point particle, which carries all characteristics of this particle with vector fields centered on this point. But all possible characteristics of all observed physical characteristics carried by this harmonic oscillating point are explained by this CAP compliant extended mathematical description. One of the main characteristics of this SR description in compliance with Einstein's CAP is the fact that the harmonic oscillating point-particle never is on its SR worldline itself, but oscillates with extremes:

$$\rho_{max} = 2\rho_{min} = \sqrt{(A/3)}\left(\frac{1}{\sqrt{f_m}} + \sqrt{f_m}\right) > 0 \tag{2.19}$$

$$2\langle\rho\rangle = \sigma(\omega,s)l_{\mathbf{h}} = \sqrt{(\mathbf{h}G/c^3)} \approx 1.61625281 \times 10^{-35} = O(10^{-35})\ [m] \tag{2.20}$$

With **h** Dirac's constant, i.e. the reduced constant of Planck, G the gravitational constant, and c the speed of light in vacuum. $\sigma(\omega, s)$ is a required energy and spin dependent constant needed to solve a matrix of more equations:

$$\sigma(\omega, s) = 2.25\mathrm{Phi}\frac{s}{t_{\mathbf{h}}} = \frac{3^2}{2^3}(\sqrt{5}+1)\frac{s}{\omega t_{\mathbf{h}}}\text{, with Planck-time } t_{\mathbf{h}} \tag{2.21}$$

This spin and energy dependent constant (2.21) yields for the force constant k:

$$k(\omega, s) = \mathrm{Phi}\frac{(\frac{2}{3}\omega)^3}{(sc)^2}\mathbf{h}\ [\mathrm{rad}^2\mathrm{kg/s}^2] \tag{2.22}$$

N.B. This solution of the force constant is ***only*** valid for the massless photon and graviton!
In the case of non-zero mass, the fraction $f_m(A,B,C)$, see (2.10), deviates from Phi and the same different constant fraction should also be inserted into (2.22).

Bosons are described with closed BC and, as a result of that, only allow interaction in the direction of motion, specified by the worldline. This is why bosons are allowed to be on the same space-time position specified by their average positions on their common worldline.

The squared solution of DE (2.8) can be solved exactly after using two BC to solve for the two consecutive first order (proper) time derivatives. In the case of fermions one can picture these BC as the value of polar distance ρ and its first time derivative ρ' after a rotation of $\Delta\varphi = 2\pi sn$ enforced to have the same value for any possible

Used work:

1. General Theory of Relativity
   P.A.M. Dirac, Princeton Landmarks in Physics. ISBN 0-691-01146-X
2. CAP (i.e. curvature) induced doubling of degrees of freedom is explained in the paper:
   http://quantumuniverse.eu/Tom/Curvature%20and%20QM.pdf .