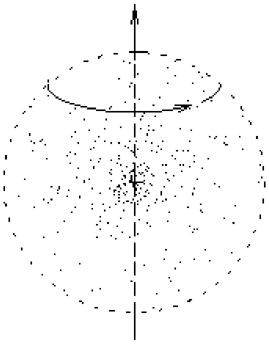

Intrinsic Angular Momentum

The following description is bogus. That it, this is not ``really'' what intrinsic angular momentum is all about; but it is possible to understand it in ``common sense'' terms, so we can use it as a mnemonic technique. Many concepts are introduced this sort of ``cheating'' until students get comfortable enough with them to define them rigourously. (The truth about **spin**, like much of *QM*, can never be made to seem sensible; it can only be gotten used to!)



Imagine a big fuzzy ball of mass spinning about an axis. While you're at it, imagine some electric charge sprinkled in, a certain amount of charge for every little bit of mass. (If you like, you can think of a cloud of particles, each of which has the same charge-to-mass ratio, all orbiting about a common axis.) Each little mass element contributes a bit of angular momentum and a proportional bit of magnetic moment, so that $\vec{\mathbf{L}} = \sum \vec{\mathbf{r}} \times \vec{\mathbf{p}}$ (summed over all the mass elements) and, as for a single particle, $\vec{\mu} =$(constant)$\times\vec{\mathbf{L}}$. If the charge-to-mass ratio happens to be the same as for an *electron*, then (constant) $= \mu_B$, the Bohr magneton.

Now imagine that, like a figure skater pulling in her/his arms to spin faster, the little bits of charge and mass collapse together, making r smaller everywhere. To conserve angular momentum (which is *always* conserved!) the momentum p has to get bigger - the bits must spin faster. The relationship between L and μ is such that μ also remains constant as this happens.

Eventually the constituents can shrink down to a *point* spinning infinitely fast. Obviously we get into a bit of trouble here with both relativity and quantum mechanics; nevertheless, this is (sort of) how we think (privately) of an *electron*: although we have never been able to find any evidence for ``bits'' within an electron, we are able to rationalize its possession of an *irreducible, intrinsic angular momentum* (or ``**spin**'') in this way.

Such *intrinsic* angular momentum is *a property of the particle itself* as well as a dynamical variable that behaves just like orbital angular momentum. It is given a special label ($\vec{\mathbf{S}}$ instead of $\vec{\mathbf{L}}$) just to emphasize its difference. Like $\vec{\mathbf{L}}$, it is *quantized* - *i.e.* it only comes integer multiples of a fundamental quantum of intrinsic angular momentum - but (here comes the weird part!) that quantum can be either $\hbar$, as for $\vec{\mathbf{L}}$, or $\frac{1}{2}\hbar$!

In the following, s is the ``spin quantum number'' analogous to the ``orbital quantum number'' ℓ such that the spin angular momentum $\vec{\mathbf{S}}$ has a magnitude $|\vec{\mathbf{S}}| = \hbar\sqrt{s(s+1)}$ \ and a z component $S_z = m_s\hbar$ where $\hat{z}$ is the chosen spin quantization axis. The magnetic quantum number for spin $\frac{1}{2}$ has only two possible values, spin ``up'' ($m_s = +\frac{1}{2}$) and spin ``down'' ($m_s = -\frac{1}{2}$). This is the explanation of the Stern-Gerlach result for silver atoms: with no orbital angular momentum at all, the Ag atoms have a single ``extra'' electron whose spin determines their overall angular momentum and magnetic moment.
